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Damping Forces of Vibrating Cylinder in Confined Viscous Fluid by a Simplified Analytical Method

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The simplified analytical solutions for viscous damping have been formulated, considering the results obtained by an existing numerical method, for relatively narrow annular configurations : (i) when a rigid cylinder executes translational oscillation in the plane of symmetry, and (ii) a flexible cylinder vibrates in its first mode as a clamped-clamped beam subject to axial flow. For narrow annular passages, the viscous damping has significant effects on fluid-dynamic forces. In such a case, an inviscid fluid model is acceptable for estimating added mass. This theory is developed for both relatively high and low oscillatory Reynolds numbers. In terms of computational efficiency, it is useful to obtain the viscous damping forces using this approximate method. Also this method has important benefit for the future study of stability analysis of system ; since, the viscous damping forces obtained by the present method can be expressed in terms of the oscillatory Reynolds number explicitly. To validate this theory, the results are compared with the ones obtained by the full viscous theories in the previous works. These results were found to be in reasonably good agreement with the results of the full theories.

Key Words : Flow-Induced Vibration, Added Mass, Viscous Damping, Penetration Depth, Oscillatory Reynolds Number

1. Introduction

When a cylinder vibrates in viscous fluid, the fluid-dynamic forces acting on the moving cylinder are influenced by the fluid properties, including axial flow velocity, and also by the geometry of the system (Chen, et al., 1976 ; Yang and Moran, 1979 and Païdoussis, et al., 1989). Fluid damping for circular cylinders in various flow conditions have been summarized by Chen (1981). In general, the resulting forces become larger with the confinement of the annulus. According to the existing results, the added mass coefficient is mainly affected by the geometry for narrow configurations, especially in the case of relatively high oscillatory Reynolds number Re_s. The effect of Re_s itself on the added mass is relatively small. Thus, the added mass for narrow annuli can be estimated by potential theories (Fritz, 1972 ; Chung and Chen, 1977). In contrast to the added mass, the damping coefficient is strongly dependent on the oscillatory Reynolds number, as well as on the geometry of system. Especially for narrow annular passages, the effect of viscous damping on the fluid-dynamic forces should be considered for the analysis of stability, even if the viscosity of the fluid is relatively small. Due to the confinement, the viscous drag force by unsteady viscous flow is an important component of the fluid forces.

It was found in the previous numerical study based on the spectral collocation method (Sim and Cho, 1993a), that higher-order terms in the Chebyshev polynomials are clearly needed, as the oscillatory Reynolds number is increased ; since, the penetration depth, defined by $\delta_p =$

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$\sqrt{2\nu/\omega}$ (Schliching, 1979), is very small compared to the annular space between the two cylinders. The penetration depth is associated with unsteady viscous wave. Thus a very large system of equations is expected to become necessary for such problems. Potential flow theory can of course be utilized to obtain the added mass, but the viscous forces cannot easily be estimated, because of the large size of the matrices obtained by viscous flow theory in the spectral collocation method. This is a reason why the approximate method for obtaining the viscous forces with axial flow was developed in the previous work (Sim and Cho, 1993b).

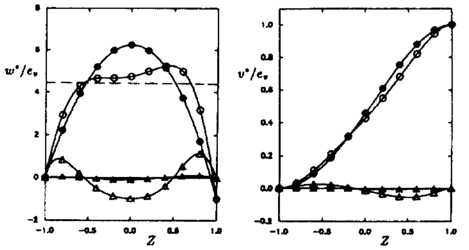
By assuming a relatively low frequency and Reynolds number, the previous analytical theory undertaken by the author for the flexural motion of cylinder subject to axial flow, has been formulated to estimate the fluid-dynamic forces. To approximate the viscous force, a parabolic radial distribution of the unsteady circumferential flow velocity in the annular flow was introduced. In the previous work devoted to the spectral method, it was found that the unsteady flow velocity profile is different from the parabolic one when the ratio of the penetration depth with respect to the annular space is relatively small. The initial motivation was to modify the analytical method for reducing the limitations mentioned above.

With the full viscous theories shown in the previous works, the damping coefficient contains the effect of the oscillatory Reynolds number, but the coefficient was expressed as implicit function of the oscillatory Reynolds number. Thus, an iteration procedure is required to obtain the eigenfrequencies of a system. In this point of view, it is convenient to obtain the damping force through an analytical method where the coefficient can be expressed as explicit function of the oscillatory Reynolds number. Using the present analytical method, the damping force can be estimated for both high and low oscillatory Reynolds numbers and expressed in terms of the oscillatory frequency and motion of the moving cylinder. This is main benefit of the present approximate method for future purpose of stability analysis.

2. Unsteady Viscous Drag Force Due to Translational Oscillation

To develop the present analytical theory, the results obtained by the previous theory(Sim and Cho, 1993a) are carefully examined. According to those results, the unsteady pressure variation across the annular gap is very small and the mean value of the unsteady flow velocity in the circumferential direction (in phase with the velocity of the moving cylinder) is approximately equal to its flow velocity obtained by the potential flow theory. Moreover, the second order term of Chebyshev polynomials defined for the circumferential flow velocity is quite large for low oscillatory Reynolds numbers, which means that the distribution of the velocity has a parabolic profile in this case. For high oscillatory Reynolds numbers, the amplitude of the circumferential-flow velocity at a penetration depth ($r=a+\delta_p$ or $r=b-\delta_p$), is almost the same as the corresponding one given by potential flow theory. In the present analysis, the oscillatory Reynolds number is expressed in terms of the radius ratio to penetration depth as $Re_s=2(a/\delta_p)^2=\omega a^2/\nu$.

Using the previous numerical methods undertaken by the author, the nondimensional flow velocities, circumferential(ω^*) and radial(v^*) components, to the lateral velocity(e_v) of the moving cylinder are plotted in Fig. 1 along the radial direction for concentric configuration. In this figure, the dotted line represents the circumferential-flow velocity obtained by the potential flow theory. As mentioned before, the shape of the real part of complex-flow velocity in the circumferential direction is similar to that for steady viscous flow (Laminar or turbulent flow); however, for unsteady flow, the characteristic of the profile is dependent on the oscillatory Reynolds number instead of Reynolds number defined for steady flow. Physically, the mean flow velocity can be estimated by potential flow theory for very high oscillatory Reynolds number since the penertra-



(a) (b)

Fig. 1 The distribution of the nondimensional amplitude of the unsteady flow velocity for $b/a=1.25$, $Re_s=50$(filled symbols) and $Re_s=1{,}740$(open symbols) across the annular space ; (a) the circumferential and (b) the radial components. Viscous theory ; ○ ●, real part ; △ ▲, imaginary part ; ---, circumferential components obtained for the potential flow (Sim and Cho. 1993a)

tion depth for the case is very small compared to the annular gap.

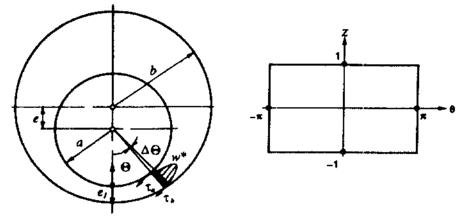
In the present analysis, the unsteady radial flow velocity is not considered, in order to simplify the problem, so that its effect on the drag forces is neglected. Physically, the unsteady skin friction on the surface is induced mainly by the unsteady circumferential flow velocity. Also, the unsteady pressure drop in the circumferential direction is affected by the skin friction. Under these considerations, a drastically simplified form of the Navier-Stokes equations is obtained ; from that starting point, the problem is formulated to evaluate the viscous drag force.

2.1 Formulation

Considering the assumptions defined for the present problem, the simplified momentum equation is obtained, by integrating the Navier-Stokes equation across the annular space for the element shown in Fig. 2, as

$$-\frac{H}{r}\frac{\partial p^*}{\partial\Theta}-\tau_a-\tau_b=\int_a^b\left[\frac{1}{r}\frac{\partial}{\partial\Theta}(\rho\omega^{*2})-\frac{\partial\omega^*}{\partial t}\right]_{\Theta+\Delta\Theta}dr-\int_a^b\left[\frac{1}{r}\frac{\partial}{\partial\Theta}(\rho\omega^{*2})-\frac{\partial\omega^*}{\partial t}\right]_{\Theta}dr, \qquad (1)$$

where the skin frictions on the surfaces of the inner and outer cylinder are given by



(a) (b)

Fig. 2 The shear stress acting on surface elements of the inner and outer cylinders due to the unsteady circumferential flow velocity and computational domain (Z, θ) obtained by the coordinate transformation

$$\tau_a=\mu\frac{\partial\omega^*}{\partial r}\bigg|_{r=a},\quad \tau_b=-\mu\frac{\partial\omega^*}{\partial r}\bigg|_{r=b}, \qquad (2)$$

where w^* and p^* denote the unsteady circumferential flow velocity and the unsteady pressure, respectively By the assumption of small amplitude motion of the cylinder in a narrow annular passage, the right hand-side of Eq. (1) can be neglected.

Introducing the dimensionless parameters, $\bar{p}=p^*/(\rho a^2\omega^2\bar{e}e^{\iota\omega t})$ and $\bar{w}=w^*/(\iota a\omega\bar{e}e^{\iota\omega t})$, with the aid of coordinate, $Z=1-2(r-a)/H$, transformation-see Fig. 2, the governing equation for a narrow annulus are nondimensionalized as

$$\frac{\partial\bar{p}}{\partial\theta}=\iota\frac{2}{Re_s}\frac{1+h/2}{h^2}\left[\frac{\partial\bar{w}}{\partial Z}\bigg|_{Z=1}-\frac{\partial\bar{w}}{\partial Z}\bigg|_{Z=-1}\right], \qquad (3)$$

and the shear stress is obtained by

$$\tau=\mu\frac{\partial w^*}{\partial r}=-\iota\rho a^2\omega^2\bar{e}e^{\iota\omega t}\frac{2}{hRe_s}\frac{\partial\bar{w}}{\partial Z}, \qquad (4)$$

where $a\bar{e}e^{\iota\omega t}$ denotes the lateral displacement of oscillatory motion of the cylinder so $\bar{e}$ is nondimensional lateral displacement, and the annular space is expressed as $H=ah(\theta)=\sqrt{b^2-e^2\sin^2\theta}-e\cos\theta-a$. Generally, the lateral displacement is considered to be small in flow-induced vibration problems so that the nonlinear effects of a production between unsteady terms in governing equation on unsteady viscous flow might be negligible.

For the purpose of this simplified analysis, the mean flow velocity $\bar{w}^*$ across the gap will be calculated by potential theory. From the velocity potential ϕ and the relationship $w^*=(1/r)(\partial\phi/\partial\Theta)$, one can obtain by integrating over the gap

$$\bar{w}^*=\frac{1}{H}\int_a^b \frac{1}{r}\frac{\partial\phi}{\partial\Theta}dr$$
$$=\frac{1}{a}\int_{-1}^{1}\frac{1}{2-h(Z-1)}L(\phi)dZ, \quad (5)$$

where the operator, $L(\phi)$, can be expressed as $L(\phi)=\partial\phi/\partial\theta+[(1-Z)/h]\cdot(\partial h/\partial\theta)\cdot(\partial\phi/\partial Z)$. Considering the results shown in Fig. 1 for potential flow, the last term of the light hand side in the equation, $L(\phi)$, is negligible especially for narrow annulus or an annulus of relatively small eccentricity.

To obtain the numerical solutions, the following expansion form was utilized

$$\phi=\iota\omega a^2 \hat{e} e^{\iota\omega t}\sum_{j=0}^{m}\sum_{k=0}^{n}\Phi_{jk}T_j(Z)F_k(\theta) \quad (6)$$

which have already been defined in the previous work. For the present problem, the Fourier function F_k may be expressed as cosine functions due to the flow symmetry with respect to the plane of eccentricity. Considering the expansion form with dimensionless parameters, the dimensionless mean flow velocity, $\hat{w}=(\iota a\omega\hat{e}e^{\iota\omega t})$, can be obtained from Eq. (5) as

$$\hat{w}=\sum_{k=0}^{n}ks(k\theta)\int_{-1}^{1}\frac{-1}{2-h(Z-1)}\sum_{j=0}^{m}\Phi_{jk}T_j(Z)dZ$$
$$=\sum_{k=0}^{n}\bar{W}_k s(k\theta), \quad (7)$$

where $s(k\theta)$ denotes $\sin k\theta$. By the potential theory based on the spectral method, the coefficients Φ_{jk} have already been obtained. Hence, the coefficients $\bar{W}_k$ can be determined.

To solve the pressure drop along the circumferential direction, the shear stress on the surface of the cylinders is considered by carefully investigating the distribution of the unsteady circumferential flow velocity across the annular space for both cases : (a) when the ratio of the penetration depth with respect to annular gap is relatively large and (b) the ratio is relatively small. This ratio is related to the oscillatory Reynolds number by the definition, $\delta_p/H=\sqrt{2/Re_s}\cdot a/H$, where δ_p is the penetration depth.

(a) case of relatively low oscillatory Reynolds number

As discussed before, the circumferential flow velocity w^* has a parabolic profile in this case. In this method, the dimensionless flow velocity $\hat{w}$ may be approximated in the following form

$$\hat{w}(z,\ \theta)=\sum_{k=0}^{n}(W_{2k}^{\dagger}Z^2+W_{1k}^{\dagger}Z+W_{0k}^{\dagger})s(k\theta), \quad (8)$$

subject to the boundary conditions

$$\sum_{k=0}^{n}(W_{2k}^{\dagger}Z^2+W_{1k}^{\dagger}Z+W_{0k}^{\dagger})s(k\theta)|_{Z=1}=-\sin\theta,$$
$$\sum_{k=0}^{n}(W_{2k}^{\dagger}Z^2+W_{1k}^{\dagger}Z+W_{0k}^{\dagger})s(k\theta)|_{Z=-1}=0.$$

Hence, the mean value of it, $\bar{\hat{w}}$, may be obtained by integrating $\hat{w}$ over the annular gap

$$\hat{w}=\frac{1}{2}\sum_{k=0}^{n}\int_{-1}^{1}(W_{2k}^{\dagger}Z^2+W_{1k}^{\dagger}Z+W_{0k}^{\dagger})s(k\theta)dZ$$
$$=\sum_{k=0}^{n}\left(\frac{1}{3}W_{2k}^{\dagger}+W_{0k}^{\dagger}\right)s(k\theta). \quad (9)$$

Taking account of two boundary conditions and of the above equation together with Eq. (7), the three-unknown coefficients could clearly be expressed in terms of $\bar{W}_k$ in the form

$$W_{2k}^{\dagger}=-\frac{3}{2}\bar{W}_k-\frac{3}{4}\delta,\ \ W_{1k}^{\dagger}=-\frac{1}{2}\delta,$$
$$W_{0k}^{\dagger}=\frac{3}{2}\bar{W}_k+\frac{1}{4}\delta\ , \quad (10)$$

where $\delta=0$ when $k\neq 1$, or $\delta=1$ when $k=1$; from which the skin friction on the surface can be obtained. In view of Eq. (3), one ought to derive the following equation from Eq. (8) with the known coefficients $W_{2k}^{\dagger}$, $W_{1k}^{\dagger}$ and $W_{0k}^{\dagger}$, to obtain the pressure distribution,

$$\left.\frac{\partial\hat{w}}{\partial Z}\right|_{Z=1}=-(3\bar{W}_k+2\delta)s(k\theta),$$
$$\left.\frac{\partial\hat{w}}{\partial Z}\right|_{Z=-1}=(3\bar{W}_k+\delta)s(k\theta). \quad (11)$$

(b) case of relatively high oscillatory Reynolds number

By inspection of the distribution of the unsteady circumferential flow velocity across the annular space for this case (see Fig. 1), the radial derivative of the velocity in phase with the velocity of the moving cylinder can be approximated in terms of the penetration depth as

$$\left.\frac{\partial w^*}{\partial r}\right|_{r=a}=\frac{\bar{w}^*+e_v\sin\Theta}{\delta_p},$$
$$\left.\frac{\partial w^*}{\partial r}\right|_{r=b}=-\frac{w^*}{\delta_p}, \tag{12}$$

through which the unsteady pressure drop shown in Eq. (1), may be determined. In the above equation, the mean flow velocity w^* has already been obtained by potential theory as shown in Eq. (5). To consider the boundary condition, on the surface of the moving inner cylinder, the lateral velocity of the moving cylinder, e_v, is added to the left side of the first equation.

Utilizing the dimensionless parameters in the transformed domain, the above equation can be rewritten in nondimensional form

$$\left.\frac{\partial \bar{w}}{\partial z}\right|_{Z=1}=-\frac{ah}{2\delta_p}\sum_{k=0}^{n}(\bar{W}_k+\delta)s(k\theta),$$
$$\left.\frac{\partial \bar{w}}{\partial Z}\right|_{Z=-1}=\frac{ah}{2\delta_p}\sum_{k=0}^{n}\bar{W}_k s(k\theta), \tag{13}$$

where the coefficients for the nondimensional mean flow velocity, $\bar{W}_k$, are defined in Eq. (7). Thus, the unsteady pressure drop and the skin friction on the surfaces of the cylinders can be estimated, considering Eqs. (1) and (2).

2.2 Unsteady drag force

Having determined skin friction on the surfaces, the unsteady pressure distribution along the circumference may now be found both for the relatively low and high oscillatory Reynolds number. Then the unsteady drag force can be obtained by considering the skin friction on the surface of the moving cylinder and the pressure drop along the circumference of the inner cylinder.

For the case (a) of low oscillatory Reynolds number, substituting Eq. (11) into Eq. (3) leads to

$$\frac{\partial \hat{p}}{\partial \theta}=-\iota\frac{12}{Re_s}\frac{1+h/2}{h^2}\sum_{k=0}^{n}(\bar{W}_k+\delta/2)s(k\theta). \tag{14}$$

Proceeding similarly with Eq. (13), for the case (b) of high oscillatory Reynolds number, the nondimensional pressure is expressed as

$$\frac{\partial \hat{p}}{\partial \theta}=-\iota\sqrt{\frac{2}{Re_s}}\frac{1+h/2}{h}\sum_{k=0}^{n}(\bar{W}_k+\delta/2)s(k\theta). \tag{15}$$

Taking account of the pressure and skin friction, the viscous drag forces may be obtained by integrating its components around the cylinder as

$$F_d=-a\int_0^{2\pi}\left(p^*\cos\Theta+\mu\left.\frac{\partial w^*}{\partial r}\right|_{r=a}\sin\Theta\right)d\Theta. \tag{16}$$

However, some manipulation is required to bring p^*, which is an implicit function of Θ, into a convenient form. In this respect, $p^*\cos\Theta$ is modified as

$$p^*\cos\Theta=\frac{d}{d\Theta}(p^*\sin\Theta)-\frac{dp^*}{d\Theta}\sin\Theta. \tag{17}$$

The integral of the first term of the right-hand side in the above equation is equal to zero. Hence, the equation of the drag force can be expressed in the nondimensional form

$$F_d=\rho a^2\omega^2 a\hat{e}e^{\iota\omega t}\int_0^{2\pi}\left(\frac{\partial \hat{p}}{\partial \theta}+\iota\frac{2}{hRe_s}\left.\frac{\partial \bar{w}}{\partial Z}\right|_{Z=1}\right)\sin\theta d\theta$$
$$=\iota\rho\pi a^2\omega^2 a\hat{e}e^{\iota\omega t}\hat{F}_d, \tag{18}$$

where $a\hat{e}e^{\iota\omega t}$ denotes the lateral displacement of oscillatory motion of the cylinder as mentioned before and $\hat{F}_d$ is the nondimensional viscous damping force.

Substituting Eqs. (11) and (14) into the above equation for case (a) leads to

$$\hat{F}_d=\hat{F}_{vp}+\hat{F}_{vs}, \tag{19}$$

where

$$\hat{F}_{vp}=\frac{-12}{Re_s}\frac{1+h/2}{h^2}(W_1+1/2),$$
$$\hat{F}_{vs}=\frac{-6}{Re_s}\frac{1}{h}(W_1+2/3), \tag{20}$$

in which the subscripts vp and vs stand for the unsteady pressure and the skin friction terms, respectively. Thus, the damping coefficient due to the viscous drag can be expressed as

$$C_{vd}=-\rho\pi a^2\omega\hat{F}_d. \tag{21}$$

Similarly, for case (b),

$$\hat{F}_{vp}=-\sqrt{\frac{2}{Re_s}}\frac{1+h/2}{h}(\bar{W}_1+1/2),$$

$$\hat{F}_{fs} = -\sqrt{\frac{1}{2Re_s}}\bar{W}_1. \quad (22)$$

Considering these results, the viscous drag force is expressed in terms of an explicit function of the oscillatory Reynolds number, since the coefficients Φ_{jk} and $\bar{W}_k$, determined by the potential theory, are dependent only on the geometry of system.

3. Unsteady Damping Forces Due to Flexible Oscillation of a Cylinder

By inspection of the results given by the full viscous theory based on the collocation finite-difference method, the added mass coefficient can be approximately calculated by potential flow theory for narrow annular configurations, since it is mainly affected by the geometry of the system. However, the damping forces are dependent on fluid properties as well as geometry. In general, it is well known that the damping force acting on a flexible cylinder subject to axial flow are decomposed into two terms : the viscous damping force due to the fluid viscosity, as shown in the previous section, and a force due to the Coriolis effect associated with the axial flow.

According to the results given in the full viscous theory (Sim and Cho, 1993b), the circumferential velocity is almost linearly dependent on the velocity of the flexible cylinder and the profile of this velocity along the radial direction is similar to that obtained in the two-dimensional problem shown in Fig. 1. Therefore, the viscous damping forces can be approximated by considering the unsteady pressure drop mainly due to the radial derivative of the unsteady circumferential flow velocity in annular flow. Hence, the viscous drag force due to flexural motion of the inner cylinder can be calculated by considering the viscous damping force F_d obtained by the approximate method for the two-dimensional problem based on the lateral displacement, $e_I(L/2,\ t)$, as

$$F_{d3} = F_d\frac{e_I(x,\ t)}{e_I(L/2,\ t)}, \quad (23)$$

where subscript 3 stands for the three-dimensional problem and the lateral displacement $e_I(x,\ t)$ of the inner cylinder is expressed in terms of eigenfunction, $\psi_1(x)$, which is the first mode expansion for clamped-clamped beam,

$$e_I(x,\ t) = E(x)e^{\iota\omega t} = a_1\psi_1(x)e^{\iota\omega t}. \quad (24)$$

Generally, the motion is expressed in terms of normal-mode expansion as

$$E(x) = \sum_k a_k\psi_k(x) = \sum_k a_k[\psi_{1k}(x) + \psi_{2k}(x)], \quad (25)$$

where ψ_{1k} and ψ_{2k} denote the trigonometric and hyperbolic components of these eigenfunctions, respectively,

$$\psi_{1k} = -\cos\beta_k x + \sigma_k\sin\beta_k x,$$
$$\psi_{2k} = \cosh\beta_k x - \sigma_k\sinh\beta_k x, \quad (26)$$

and $\sigma_k = (\cosh\beta_k L - \cos\beta_k L)/(\sinh\beta_k L - \sin\beta_k L)$, the $\beta_k L$ being the corresponding eigenvalues of a clamped-clamped beam.

Thus, the viscous damping forces can be rewritten in the following form

$$F_{d3} = \iota\rho\pi a^2\omega^2 a\hat{e}(L/2)e^{\iota\omega t}\hat{F}_{d3}, \quad (27)$$

where $\hat{e} = e_I/(ae^{\iota\omega t})$ and

$$\hat{F}_{d3} = \hat{F}_d\frac{e_I(x,\ t)}{e_I(L/2,\ t)},$$

in which $\hat{F}_d = \hat{F}_{vp} + \hat{F}_{vs}$ has already been obtained in the previous section as function of the oscillatory Reynolds numbers.

Taking account of the potential flow theory based on the slender body assumption (Lighthill, 1960), the damping force due to the axial flow, which is related to the Coriolis force, may be estimated by the following equation

$$F_s = -2\rho\pi a^2\bar{U}\frac{\partial^2 e_I(x,\ t)}{\partial x\partial t}\chi, \quad (28)$$

where the mean axial flow velocity $\bar{U}$ is obtained by integrating the axial flow velocity over the annular space and the ratio of confinement is equal to the added mass coefficient determined by the slender body potential flow theory, as $\chi = (b^2 + a^2)/(b^2 - a^2)$.

Utilizing the normal mode expansion for the flexural motion, the equivalent Coriolis force can be expressed as

$$F_c = \iota\rho\pi a^2\omega^2 a\hat{e}e^{\iota\omega t}\hat{F}_c, \quad (29)$$

where

$$F_c=-2\frac{\bar{U}\beta_1}{\omega}\chi\frac{\psi_1'(x)}{\psi_1(L/2)},$$

in which the prime denotes differentiation with respect to x, and $\beta_1 L$ represents the eigenvalue for the first mode of the flexible cylinder.

Considering the viscous drag force and the equivalent Coriolis force, the total damping force acting on the flexible cylinder subject to steady axial flow in a narrow annulus can be calculated approximately by

$$F_{dt}=\iota\rho\pi a^2\omega^2 a\bar{e}e^{\iota\omega t}(\hat{F}_{d3}+\hat{F}_c)$$
$$=\iota\rho\pi a^2\omega^2 a\bar{e}e^{\iota\omega t}\hat{F}_{dt}. \quad (30)$$

4. Results and Discussion

The main purpose of the present work is to develop semi-analytical methods for estimating the damping force as influenced by the oscillatory Reynolds number, the geometry of system and the Reynolds number. To validate the present methods, the present results are compared to the results given by the viscous theories (Sim and Cho, 1993a and 1993b) based on the spectral method.

When the inner cylinder executes translational motion in the plane of symmetry, the calculations have been conducted for the cases of various ratios of the radii, b/a, with a selected oscillatory Reynolds number, rather than attempting an exhaustive parametric study. In order to investigate the effect of axial flow on the damping force acting on the inner cylinder in a narrow annulus, the damping force has been calculated with the chosen Reynolds number based on hydraulic diameter $2ha$.

In this study, we are mostly concerned with forces on the centrebody per unit length and their variation with length. With regard to the Coriolis force, it should be remarked that the integrated force (for $x\in(0, L)$) is zero -- although its variation with x is of interest and will be shown in the results that follows.

The viscous drag force with increasing oscillatory Reynolds number will be discussed for $b/a=1.25$ and $b/a=1.4$. The viscous drag coefficient $\hat{F}_d$ obtained for both cases of low and

Table 1 Comparison of the drag coefficients $\hat{F}_d$ obtained by the approximate method developed for (a) low and (b) high oscillatory Reynolds number and by the numerical method with various ratios of the penetration depth to the annular space

b/a	Re_s	δ_p/H	Approximate Result $\hat{F}_d$ (a)	Approximate Result $\hat{F}_d$ (b)	Numerical Results $\hat{F}_d$
1.25	5000	0.08	0.22	0.52	0.53
	1000	0.18	1.08	1.16	1.46
	500	0.25	2.17	1.65	2.44
	100	0.57	10.8	3.68	11.0
	50	0.80	21.7	5.21	22.1
1.4	5000	0.05	0.06	0.26	0.25
	500	0.16	0.64	0.82	0.96
	100	0.36	3.18	1.84	3.34
	50	0.50	6.35	2.60	6.61

high oscillatory Reynolds number is shown in Table 1, where the results are compared to the corresponding numerical results based on the spectral collocation method. The drag coefficient $\hat{F}_d$ is the same definition to the imaginary one $\Im(\hat{F})$ defined in the previous numerical method.

From the results, it is found that the transition region, where both approximate methods give approximately same value, is situated around $\delta_p/H=0.2$: Above this ratio of penetration depth, the method developed for low oscillatory Reynolds numbers can be used, while the other one is more suitable for high Re_s. Thus, it is true that the viscous drag force is dependent on the ratio of the penetration depth to the annular space, which affects the circumferential flow velocity profile in the radial direction.

In Fig. 3, the nondimensional damping forces for various oscillatory Reynolds numbers ($Re_s=50$, 500, 5,000 and 50,000) are illustrated for concentric annular configurations to show the effect of b/a. The results denoted by close symbols represent the nondimensional force $\hat{F}_{vp}$

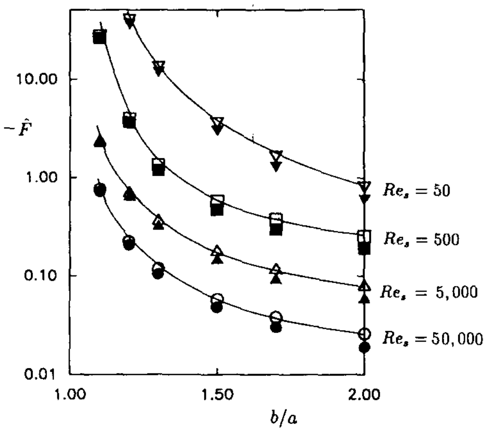


Fig. 3 Nondimensional viscous damping force obtained by the approximate method for the translational motion of the inner cylinder in concentric configurations ; considering only unsteady pressure(closed symbols), and unsteady shear stress and pressure(open symbols).——, numerical results obtained with the spectral method (Sim and Cho, 1993a)

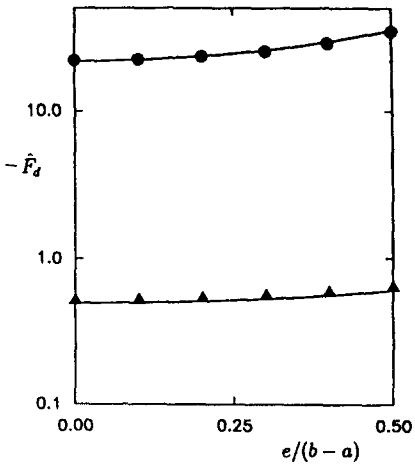
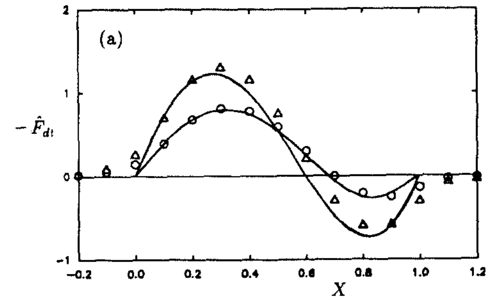


Fig. 4 Effect of annular eccentricity on the viscous damping force obtained by the approximate method for the translational motion of the inner cylinder and for $b/a=1.25$: ●, $Re_s=50$; ▲, $Re_s=5{,}000$. Compared with the numerical results obtained with the spectral collocation method (Sim and Cho, 1993a)

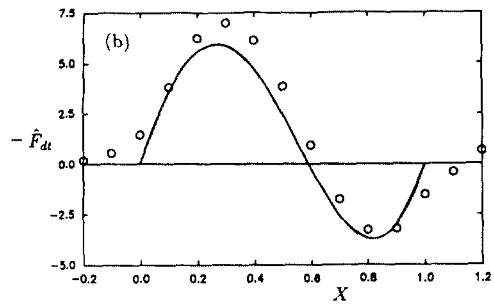
obtained by considering only the unsteady pressure. The overall results $\hat{F}_d$, including the effect of skin friction, are denoted by the open symbols. According the results, the effect of skin friction is relatively small ; however, the relative magnitude of the effect of the skin friction versus the unsteady pressure becomes larger with increasing the radius ratio b/a. By inspection of Eqs. (20) and (22), this can be expected : the ratio between the two results is of the order of h. As compared to the numerical results obtained by viscous flow theory with the spectral method (Sim and Cho, 1993a), good agreement is found between these results and the numerical results.

When the inner cylinder has translational motion in the plane of axis symmetry in an eccentric annulus, the nondimensional overall drag force $\hat{F}_d$ is presented in Fig. 4 for $b/a=1.25$, with oscillatory Reynolds numbers (a) $Re_s=50$ and (b) $Re_s=5{,}000$. For the former case, the calculation is done by the method developed for low oscillatory Reynolds numbers and for the latter case by the method developed for high oscillatory Reynolds numbers. Its results are compared to the numerical solutions. The present approximate method (developed in Section 2) is found to be adequate.

In Fig. 5, the overall damping force including the effect of the axial flow ($Re=626$ and 1,256) given by the present approximate method (developed in Section 3) are compared to the numerical results (Sim and Cho, 1993b) in case of $b/a=1.25$. In the present analysis, Reynolds number is defined by $Re_s=\bar{U}2ha/\nu$. As shown in the figure, the nondimensional damping force may be decomposed into two components for the given flexural motion (the first mode vibration as a clamped-clamped beam) ; (i) the symmetric component with respect to the middle point $X=x/L=1/2$, which is related to the unsteady viscous drag force, and (ii) the antisymmetric one, which associated with the Coriolis force (axial flow effect). It is shown in Fig. 5(a) that the antisymmetric component becomes large with increasing the Reynolds number. The damping force, containing the effect of the unsteady viscous drag and the axial flow, predicted by the approximate method agrees well with the numerical



(a) $Re_s=5{,}000$



(b) $Re_s=500$

Fig. 5 Nondimensional viscous damping force obtained by the approximate method,——; and by the collocation finite-difference method (Sim and Cho, 1993b), ○, $Re=626$; △, $Re=1{,}256$, for the first-mode flexural oscillations of the inner cylinder :

ones, which means the full viscous theory shown in the previous work is validated indirectly, however its comparison have been conducted in special case – slender cylinder subject to narrow annular flow. The effects of the oscillatory motion on the fluid-dynamic forces at upstream and downstream are not shown in the present results ; however, the effects were predicted by the previous numerical theory.

Taking account of the above results, the interesting remarks are as follows ; (i) the present approximate method can be utilized for estimating the damping force, especially for narrow configuration where the damping force has important role on the dynamics of system and the virtual mass can be estimated by potential theory ; (ii) the damping force can be expressed in term of the circular frequency of the moving cylinder explicitly through the approximate method – this expression is very convenient to analyse the stability of system ; (iii) the unsteady viscous drag force is proportional to $1/Re_s$ for relatively low oscillatory numbers and to $1/\sqrt{Re_s}$ for relatively high ones.

References

Chen, S. S., Wambsganss, M. W. and Jendrzejczyk, J. A., 1976, "Added Mass and Damping Vibrating Rod in Confined Viscous Fluid," *Journal of Applied Mechanics,* Vol.43, pp. 325～329.

Chen, S. S., 1981, "Fluid Damping for Circular Cylindrical Structures," *Nuclear Engineering and Design,* Vol.63(1), pp. 81～100.

Chung, H. and Chen, S. S., 1977, "Vibration of a Group of Circular Cylinders in a Confined Fluid," *Journal of Applied Mechanics,* Vol.44, pp. 213～217.

Fritz, R. J., 1972, "The Effect of Liquids on the Dynamic Motions of Immersed Solids," ASME *Journal of Engineering for Industry,* Vol.94, pp. 167～173.

Lighthill, M. J., 1960, "Note on the Swimming of Slender Fish," *Journal of Fluid Mechanics,* Vol.9, pp. 305～317.

Päidoussis, M. P., Mateescu, D. and Sim, W.-G., 1990, "Dynamics and Stability of a Flexible Cylinder in a Narrow Coaxial Cylindrical Duct Subjected to Annular Flow," *Journal of Applied Mechanics,* Vol.57, pp. 232～240.

Schliching, H., 1979, "Boundary-Layer Theory," *McGraw-Hill* Book Co., 7th Edition, New York.

Sim, W.-G. and Cho, Y.-C., 1993a, "Unsteady Potential and Viscous Flows between Eccentric Cylinders," *KSME Journal,* Vol.7 (1), pp. 55～69.

Sim, W.-G. and Cho, Y.-C., 1993b, "Study of Unsteady Fluid-Dynamic Forces Acting on a Flexible Cylinder in a Concentric Annulus," *KSME Journal,* Vol.7 (2), pp. 144～157.

Yang, C. I. and Moran T. J., 1979, "Finite-Element Solution of Added Mass and Damping of Oscillation Rods in Viscous Fluids," *Journal of Applied Mechanics,* Vol.46, pp. 519～523.